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Large-N bounds on, and compositeness limit of gauge and gravitational interactions

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Abstract: In a toy model of gauge and gravitational interactions in $D \geq 4$ dimensions, endowed with an invariant UV cut-off Λ, and containing a large number N of non-self-interacting matter species, the physical gauge and gravitational couplings at the cut-off, $\alpha_g \equiv g^2 \Lambda^{D-4}$ and $\alpha_G \equiv G_N \Lambda^{D-2}$, are shown to be bounded by appropriate powers of $1/N$. This implies that the infinite-bare-coupling (so-called compositeness) limit of these theories is smooth, and can even resemble our world. We argue that such a result, when extended to more realistic situations, can help avoid large-N violations of entropy bounds, solve the dilaton stabilization and GUT-scale problems in superstring theory, and provide a new possible candidate for quintessence.

Keywords: Renormalization Regularization and Renormalons, 1/N Expansion, Models of Quantum Gravity.

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1. The toy model and the claim

Consider a toy model of gauge and gravitational interactions in $D \geq 4$ space-time dimensions, minimally coupled to a large number of spin 0 and spin 1/2 matter fields. Let us endow the model with a cut off Λ, assumed to be finite and to preserve gauge invariance and general covariance. The toy model is supposed to mimic a bona-fide higher dimensional UV-finite theory of all interactions such as those provided by superstring theory. Let us also neglect, for the moment, matter self interactions. The tree-level action of the model thus reads, in obvious notations

$$S_0 = -\frac{1}{2}\int d^D x\sqrt{-g}\left[\kappa_0^{-2}R + \frac{1}{2}g_0^{-2}\sum_{k=1}^{N_1} F^k_{\mu\nu}F^{k\mu\nu}\right] + \sum_{i=1}^{N_0}\left((D_\mu\phi)_i(D^\mu\phi)_i + m_0^2\phi_i^2\right) + \\ + \int d^D x\sqrt{-g}\left[\sum_{j=1}^{N_{1/2}}\left(\bar{\psi}_j(i\gamma\cdot D\psi)_j + m_{1/2}\bar{\psi}_j\psi_j\right) + \cdots\right], \tag{1.1}$$

where dots stand for an ultraviolet completion of the model implementing the UV cutoff. We have given for simplicity a common mass m_0 to all the spin zero fields and a common mass $m_{1/2}$ to all spin 1/2 fields. These masses are assumed to be small compared to the UV cut-off Λ.

We are interested in the value of the renormalized gauge and gravitational couplings, g^2 and $\kappa^2 = 8\pi G_N$, as a function of their bare values, g_0^2 and κ_0^2, and of Λ, when the total number of matter fields $N = N_0 + N_{1/2} \to \infty$, while their relative ratios are kept fixed. We claim that, in the above-defined model, and modulo a certain generic assumption about one-loop contributions, the following bounds hold:

$$\alpha_g \equiv g^2\Lambda^{D-4} < \frac{c_1}{N^p}, \qquad D > 4, \\ \alpha_G \equiv G_N\Lambda^{D-2} < \frac{c_2}{N}, \qquad D \geq 4, \tag{1.2}$$

where c_1, c_2 are positive constants (typically smooth functions of the relative abundances N_i/N) to be computed at the one-loop level, and p is a number between 0 and 1. The case of the gauge coupling at $D = 4$ needs a separate discussion because of infrared effects.

We also claim that both bounds are saturated in the compositeness (infinite-bare-coupling) limit and, therefore, that such a limit exists, is smooth, and can possibly be a realistic one, in agreement with older proposals [1].

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2. Proving the claim

Consider the Feynman path integral corresponding to the action (1.1) and integrate out completely (i.e. on all scales) the matter fields. Since they only appear quadratically the exact result is:

$$\begin{aligned} I &= \int dg_{\mu\nu} dA_\mu^k d\phi_i d\psi_j \exp\left(i(S_{0,\text{gravity}} + S_{0,\text{gauge}} + S_{0,\text{matter}} + \ldots)\right) \\ &= \int dg_{\mu\nu} dA_\mu^k \exp(iS_{\text{eff}})\,, \end{aligned} \tag{2.1}$$

where

$$S_{\text{eff}} = S_{0,\text{gravity}} + S_{0,\text{gauge}} - \frac{1}{2}\operatorname{tr}\log\nabla^2(g,A) + \operatorname{tr}\log(\gamma\cdot D(g,A))\,, \tag{2.2}$$

and the trace includes the sum over the representations to which the matter fields belong, in particular a sum over "flavour" indices.

The latter two terms in S_{eff} can be evaluated by standard heat-kernel techniques [2]. The result is well known to contain local as well as non-local terms. In $D > 4$ the non-local terms start with at least four derivatives. In $D = 4$ this is still true for the gravity part but non-local contributions appear already in the F^2 terms of the gauge-field action. Thus we write:

$$S_{\text{eff}} = S_{0,\text{gravity}}\left(1 + (c_0 N_0 + c_{1/2}N_{1/2})\kappa_0^2\Lambda^{D-2}\right) + S_{0,\text{gauge}}\left(1 + g_0^2(\beta_0 + \beta_{1/2})\frac{\Lambda^{D-4}}{D-4}\right) + S'\,, \tag{2.3}$$

with the following explanatory remarks. S' contains higher derivative (and generally non local) terms. The other terms in the gauge plus gravity effective action are local with the already mentioned exception of the gauge kinetic term in $D = 4$: this is indicated symbolically by the presence of a pole at $D = 4$, to be explained better below. Note that the corrections to the tree-level action are independent of the bare gauge and gravitational couplings (the explicit factors appearing in (2.3) being canceled by those implicit in the definition of the tree-level actions). Finally, the constants $c_0, c_{1/2}, \beta_0, \beta_{1/2}$ are in principle computable in any given theory; their order of magnitude in the large N limit will be discussed below.

We now have to discuss the effect of including gauge and gravity loops or, if we prefer, to complete the functional integral by integrating over $g_{\mu\nu}$ and A_μ^k after having introduced suitable sources. This is, in general, quite non-trivial, however appropriate large-N limits can help. Let us start with the effect of these last integrations on the gauge kinetic term, i.e. with the renormalization of the gauge coupling due to gravity and gauge loops. Obviously, such a renormalization adds to the one due to matter loops, and already included in (2.3). If we consider a large-N limit such that, not only $N_0, N_{1/2} \to \infty$, but also $\beta_0 + \beta_{1/2} \to \infty$, the effective coupling after matter-loop renormalization is arbitrarily small. In this case, the one-gauge-loop contribution dominates the remaining functional integrals (the theory having become almost classical) and we get for the final low-energy gauge effective action

$$\Gamma_{\text{eff}}^{\text{gauge}} = -\frac{1}{4}\int\sqrt{-g}\left[g_0^{-2} + (\beta_0 + \beta_{1/2})\frac{\left(\Lambda^{2\epsilon} - (q^2+m^2)^\epsilon\right)}{\epsilon} - \beta_1\frac{\left(\Lambda^{2\epsilon} - (q^2)^\epsilon\right)}{\epsilon}\right]F_{\mu\nu}^2, \tag{2.4}$$

where $\epsilon = (D-4)/2$ and, for the sake of notational simplicity, we have taken $m_0 = m_{1/2} = m$.

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In order for our approximations to be justified we need to argue that the quantity $(\beta_0 + \beta_{1/2})$ is sufficiently large and positive, indeed that it is parametrically larger than the gauge field contribution β_1. The latter is proportional to the quadratic Casimir of the adjoint representation C_A. Recalling the way matter fields couple to gauge fields, we find that this condition is satisfied provided $\beta_0 + \beta_{1/2} \sim C_M N_f \frac{d_M}{d_A} \gg C_A$, where d_M, C_M represent dimensionality and quadratic Casimir for the matter representation M, respectively. In a QCD-like theory with gauge group $\mathrm{SU}(N_c)$ this would correspond to a large N_f/N_c ratio.

What happens to the effective theory at low energy depends very much on whether D is larger or equal to 4. If $D > 4$, $\Gamma_{\text{eff}}^{\text{gauge}}$ is local and we can neglect the corrections proportional to m^2 or q^2. The renormalized gauge coupling is bound, at all scales, i.e.

$$4\pi\alpha_g \equiv g^2\Lambda^{D-4} \sim \left[g_0^{-2}\Lambda^{4-D} + (\beta_0 + \beta_{1/2}) - \beta_1\right]^{-1} \leq (\beta_0 + \beta_{1/2} - \beta_1)^{-1}, \qquad D > 4, \tag{2.5}$$

and the upper bound is reached at infinite bare coupling, $g_0^{-2} \to 0$. Note that this conclusion only holds "generically" i.e. under the assumption that no cancellation between β_0, $\beta_{1/2}$ and β_1 prevents their combination appearing in (2.5) from growing large and positive (the relative signs appearing there are a matter of conventions, the actual signs being dependent of the explicit implementation of the cut-off)

If $D = 4$ the situation is more complicated and interesting. The poles in (2.4) at $D = 4$ have to be interpreted as infrared logarithms containing in their argument the box operator (as well as the mass for the matter contribution). They provide the well-known logarithmic running of the gauge couplings. In this case the bound on the gauge coupling is scale dependent and reads:

$$\begin{aligned} 4\pi\alpha_g(q) &\sim \left[g_0^{-2} + (\beta_0 + \beta_{1/2})\log\left(\frac{\Lambda^2}{(q^2+m^2)}\right) - \beta_1 \log\left(\frac{\Lambda^2}{q^2}\right)\right]^{-1} \\ &\leq \left[(\beta_0 + \beta_{1/2})\log\left(\frac{\Lambda^2}{(q^2+m^2)}\right) - \beta_1 \log\left(\frac{\Lambda^2}{q^2}\right)\right]^{-1}, \qquad D = 4, \end{aligned} \tag{2.6}$$

and, again generically, the limit is reached at infinite bare coupling. The physical meaning of (2.6) is quite clear. Above the matter scale m the gauge theory is weakly coupled at large-N. However, below that scale, the matter fields do no longer contribute to the running and, for the non-abelian case, the theory evolves towards strong coupling in the IR. It is quite easy to estimate the confinement scale of the theory at infinite bare coupling,

$$\frac{\Lambda_{\text{conf}}}{\Lambda} = \left(\frac{m}{\Lambda}\right)^{1+\beta_{\text{tot}}/\beta_1}, \tag{2.7}$$

where $\beta_{\text{tot}} = (\beta_0 + \beta_{1/2}) - \beta_1$ is the total one-loop β-function coefficient (which is positive and large in the limit we consider). Note that eq. (2.7) can lead easily to exponentially large scale hierarchies. It would be interesting to try and verify (2.7) by numerical simulations on scalar QCD with a small gauge group (say SU(2)) and a large number of flavours, and work along these lines is in progress. Note also that the logarithmic terms appearing in (2.6) should be actually accompanied by "threshold" corrections, i.e. by a scale-independent

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renormalization that would also affect the value of the renormalized gauge coupling at the cut-off. Like the terms appearing in (2.5), these finite corrections depend on the actual implementation of the cut-off, but, quite generically, we expect their contribution to $\alpha_g^{-1}(\Lambda)$ to be $O(\beta_0 \pm \beta_{1/2} \pm \beta_1)$.

We now turn to the renormalization of the Newton constant. Here the situation is the same at all $D \geq 4$. The low-energy action is local and, since gravity couples equally to all fields, the constants $c_0, c_{1/2}$ are just some calculable N-independent numbers. Since the quantity $(c_0 N_0 + c_{1/2} N_{1/2})$ is assumed to be parametrically large, graviton-loop contributions are subleading at large-N. The gauge field contribution, given the fact that it is dominated by large virtual momenta, can be estimated accurately in the one-loop approximation so that, the final result is:

$$\Gamma_{\rm eff}^{\rm gravity} = -\frac{1}{2}\int \sqrt{-g}\left[\kappa_0^{-2} + (c_0 N_0 + c_{1/2} N_{1/2} + c_1 N_1)\Lambda^{D-2}\right] R\,, \qquad (2.8)$$

where N_1 is the total number of gauge bosons (the dimensionality of the adjoint representation) and c_1 is also a calculable number of $O(1)$.

We thus obtain, at all scales, the bound on the effective Newton constant[1]

$$8\pi\alpha_G \equiv \kappa^2\Lambda^{D-2} \sim \left[\kappa_0^{-2}\Lambda^{2-D} + (c_0 N_0 + c_{1/2} N_{1/2} + c_1 N_1)\right]^{-1} \leq (c_0 N_0 + c_{1/2} N_{1/2} + c_1 N_1)^{-1}, \qquad (2.9)$$

where, once more, the bound is saturated in the compositeness limit, $\kappa_0 \to \infty$.

We can finally rewrite both bounds (2.5) and (2.9) in the following suggestive way:

$$\alpha_g \leq \left\langle N_f C_M \frac{d_M}{d_A} + C_A \right\rangle^{-1}, \qquad \alpha_G \leq \langle N_f d_M + d_A \rangle^{-1} \qquad (2.10)$$

where $\langle \ldots \rangle$ denotes an average over all matter fields.

Before turning our attention to physical applications of our claims we have to add a word of caution about the dependence of the bounds from the details of the UV cutoff. Since the effects that we are interested in come from large virtual momenta in the loops, it is clear that the UV completion of the model can influence the final result. Work is in progress [5] to verify under which conditions the claims of this paper are supported by explicit superstring theory calculations. Furthermore, like our toy model, the superstring model should contain a parametrically-large and adjustable number of matter fields: such a possiblility naturally arises in backgrounds that include a stack of N parallel D branes and, correspondingly, SU(N) gauge fields.

3. Possible physical applications

Let us assume that the results obtained in the two preceeding sections extend in the presence of matter self-interactions, leaving a detailed analysis of this assumption to future work, and discuss some possible physical applications.

[1] Similar statements on the large-N limit of gravity were made long ago by Tomboulis [3] and, more recently, were used in the context of black-hole physics by several authors [4].

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The first is related to recent discussions of entropy bounds. It is well known (see e.g. [6]) that holographic bounds on entropy, as well as any other bounds involving the Planck length are easily threatened in the presence of a large number N of species. This is because those bounds, being themselves geometrical, do not depend *explicitly* on N, while the actual value of the entropy does increase with N. However, since the bounds scale like an inverse power of the Planck length, they can be saved if the latter decreases sufficiently fast with N. Let us consider, in particular, the example of the Causal Entropy Bound (CEB) [7], recently discussed [8] in connection with explicit, small-coupling, large-temperature calculations in CFTs [9]. It was found [8] that CFT entropy both at small and, for CFTs with AdS duals, at strong coupling, fulfills CEB provided:

$$\left(\frac{T}{M_P}\right)^{(D-2)} < \frac{c}{N} \tag{3.1}$$

Clearly, if the maximal temperature is identified with M_P, this inequality is in trouble at large-N. However, in a cut-off theory of gravity, the maximal temperature should rather be identified with the UV cutoff Λ (cf. Hagedorn's temperature in string theory) and, in that case, the inequality (3.1) just becomes, at any D, our bound (1.2). The bound (1.2) also avoids the potential problems, pointed out in ref. [10], with gravitational instability against formation of black holes from quantum fluctuations in finite-size regions.

The second possible use of the bounds (1.2) concerns the dilaton stabilization and GUT-scale problems in string theory. At tree-level, the gauge coupling and the ratio of the string (cutoff) scale to the Planck mass are given by the expectation value of a scalar field, the dilaton. Perturbatively, the dilaton's VEV is undetermined, since the dilaton acquires no potential/mass. This leads both to possible large violations of the EP [11], and to a possibly large space-time dependence of fundamental constants, such as α. Furthermore, the ratio of the GUT scale to the Planck mass tends to come out too large [12], in perturbative heterotic theory. The first two problems can be solved if non-perturbative effects induce a dilaton potential that provides the dilaton with a mass and freezes it together with the gauge and gravitational coupling. The problem with GUT and Planck scales can be probably solved [13] by going to strong string coupling (i.e. to 11-dimensional supergravity) while keeping the $4D$ couplings small.

It looks highly improbable, however, that dilaton stabilization can be achieved at weak $4D$ coupling, since any non-perturbative potential falls to zero asymptotically (at zero coupling). Strong-weak S-duality makes stabilization very unlikely also in cases where a strong $4D$ coupling regime can be replaced by a weak coupling one in a dual description. It looks therefore that dilaton stabilization may occur either near the self-dual value, ($S \sim 1$), where S-duality is of no help [14], or at strong bare $4D$ coupling, if this case cannot be mapped into a weak-coupling situation. At first sight, however, either solution of the dilaton stabilization problem would lead to unphysical values for the unified gauge coupling and for the ratio $M_P/M_{\rm GUT}$.

Our point is that these problem can be avoided if the tree-level value of M_P/M_s is renormalized down to a value $O(1/N)$ by loop effects. Similarly, $\alpha_{\rm GUT}$ gets renormalized downward, from a large tree-level value to $\alpha_{\rm GUT} \sim \langle N_f\, C_M \frac{d_M}{d_A} + C_A\rangle^{-1}$. For large-rank gauge groups (like E_6) and/or large matter representations this could easily give an acceptable value for the renormalized gauge coupling. Furthermore, given the fact that, typically, $\langle N_f\, C_M \frac{d_M}{d_A} + C_A\rangle / \langle N_f d_M + d_A\rangle \sim (C_A, C_M)/d_A$ one would find, as an order-of-magnitude relation,

$$\frac{M_P^2}{M_{\rm GUT}^2} \sim N \sim \alpha_{\rm GUT}^{-2}\,, \tag{3.2}$$

in close similarity with the open-string relation that is known to provide a much better agreement between $M_{\rm GUT}$ and M_s. Finally, a decoupling mechanism similar to the one proposed in [15] would be operational near these non-trivial minima of the dilaton potential. Notice that, form this point of view, a self-dual value of S or an infinite bare coupling ($S \to 0$) give essentially the same physics.

As a final application of this picture, I will mention the recently investigated possibility that a runaway dilaton may play the role of quintessence [16]. For this to work we have to assume that, besides automatically decoupling from baryonic matter in the strong-coupling limit, the dilaton retains, in the same limit, a non-trivial coupling to dark matter. The model naturally leads [17] to acceptable, though not necessarily unobservable, violations of the equivalence principle (in particular of universality of free fall) and to tiny (and probably unobservable) variations of the fundamental “constants”.

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